

Chapter 14

Confidence Intervals: The Basics

Key Ideas

Simple conditions for inference about an unknown population mean:

1. The sample is an SRS from the population of interest.
2. The population distribution of the variable is perfectly Normal ($N(\mu, \sigma)$).
3. The population mean μ is unknown but the population standard deviation σ is known.

- **A level C confidence interval for the mean μ of a normal population with standard deviation σ , based on an SRS of size n , is given by**

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$$

- The critical value z^* is
 - chosen so that the standard normal curve has area C between $-z^*$ and z^* ,
 - found using technology (e.g. Rossman/Chance applet) with $\frac{1-C}{2}$ as the "upper tail area."

Note: The three simple conditions above must be satisfied for the interval to be valid.

Tips for interpretation of confidence intervals:

- A level C confidence interval covers the unknown mean μ with probability C . That is, this method is "correct" $C\%$ of the time.
- Any specific confidence interval computed from a sample may or may not capture the unknown mean μ .

Sample Size for Desired Margin of Error

- The level C confidence interval for the mean of a Normal population will have a specified margin of error m when the sample size is

$$n = \left(\frac{z^* \sigma}{m} \right)^2$$

where z^* is the level C critical value of the standard Normal distribution.

Four-Step Process for Confidence Intervals

1. **State:** What is the *practical question* that requires estimating a parameter?
2. **Formulate:** *Identify the parameter* and choose a level of confidence.
3. **Solve:** Two-phased approach:
 - Check the conditions
 - Calculate the confidence interval
4. **Conclude:** Interpret your interval in the context of the *practical question*.