Chapter 15
Tests of Significance: The Basics
Key Ideas

> **"An outcome that would rarely happen if a claim were true is good evidence that the claim is not true."**
>
> **Moore p. 363**

- **Null Hypothesis** H_0: the statement being tested in a statistical test.
  - It is typically the statement we hope to reject.
  - It is typically a statement of 'no effect' or 'no difference'.

- **Alternative Hypothesis** H_a: the statement we expect or hope to be true.
  - It is typically the research hypothesis.
  - It is the statement we hope to find evidence *for*.

- **Points to note:**
    - Hypotheses are *statements* about the *population.*
    - A *statistic* should <u>never</u> appear in a hypothesis statement; only *parameters* should appear.
- ***P*-value:** The probability, computed assuming that H_0 is true, that the observed outcome would take a value as extreme or more extreme than that *actually* observed.
    - The smaller the *P*-value, the stronger the evidence against H_0 provided by the data.
    - Smaller *P*-values are evidence against H_0, because they say that the observed result is unlikely to occur when H_0 is true. Large *P*-values fail to give evidence against H_0.

- **Guidelines for interpreting *P*-values** when wording conclusions from a test of significance:
    - $.05 \leq P \leq .10$ ➔ *slight* evidence
    - $.01 \leq P \leq .05$ ➔ *some* or *moderate* evidence

    $P \leq .01$ ➔ *strong* or *convincing* evidence

The essential reasoning of a significance test:

> **"Suppose for the sake of argument that the null hypothesis is true. If we repeated our data production many times, would we often get data as inconsistent with H_0 as the data we actually have? If the data are unlikely when H_0 is true, they provide evidence against H_0."**
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> **Moore, p. 381**

z test for a Population Mean

z test for $H_0 : \mu = \mu_0$:

1. **State** the problem in context
2. **Formulate**
   a. Define the parameter of interest
   b. State H_0 and H_a.

3. **Solve**

    a. Check that the simple conditions for inference about a mean (Moore, p. 344) are satisfied.

    b. Calculate the one-sample *z* statistic

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

    c. Compute the *p*-value using technology (e.g. Rossman-Chance applet):

    - $H_a : \mu > \mu_0 \Rightarrow p - \text{value} = P(Z \geq z)$
    - $H_a : \mu < \mu_0 \Rightarrow p - \text{value} = P(Z \leq z)$
    - $H_a : \mu \neq \mu_0 \Rightarrow p - \text{value} = 2P(Z \geq |z|)$

    where "$P(Z \geq z)$" represents the area under the standard Normal curve to the right of "z", the computed value of the *z*-statistic. This probability may be computed using the Rossman-Chance applet.

4. **Conclude** in the context of the **stated** problem.

- **Significance level (α) of a test:** a pre-set cutoff level of the *p*-value:
    - *P*-value $\leq \alpha$ implies the test **is** significant at the α level.
    - *P*-value $> \alpha$ implies the test **is not** significant at the α level.

- ***Duality*** **between 2-sided significance tests and confidence levels**:
    - A significance level α (say, .05) two-sided test rejects a hypothesis $H_0 : \mu = \mu_0$ exactly when μ_0 falls outside a level $1-\alpha$ (say, .95) confidence interval for μ.

- **Differences between confidence intervals and significance tests:**
    - A confidence interval gives a range of plausible values of μ.
    - A significance test describes the plausibility of <u>one specific value</u> of μ (through the p-value).