Chapter 18: Inference about a Population Mean: Key Ideas

Conditions for inference about a mean

- The sample is an SRS
- The data have a $N(\mu,\sigma)$ distribution (symmetric and single-peaked).
- Both μ and σ are *unknown*.

Standard error

- Estimate of variability (standard deviation) of a statistic based on sample data
- Standard error of $\bar{X}$ is $\frac{s}{\sqrt{n}}$

t test for $H_0 : \mu = \mu_0$:

1. **State** the problem in context

2. **Formulate**

   a. Define the parameter of interest

   b. State H_0 and H_a.

3. **Solve**

   a. Check that the conditions are satisfied.

   b. Calculate the one-sample t statistic

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

   c. Find the p-value using the t distribution on n-1 degrees of freedom (using the t-applet).

      - $H_a : \mu > \mu_0 \Rightarrow p - \text{value} = P(T \geq t)$
      - $H_a : \mu < \mu_0 \Rightarrow p - \text{value} = P(T \leq t)$
      - $H_a : \mu \neq \mu_0 \Rightarrow p - \text{value} = 2P(T \geq |t|)$

4. **Conclude** in the context of the **stated** problem.

t confidence interval for μ:

1. **State** the parameter estimation problem in context
2. **Formulate**
    a. Define the parameter of interest
    b. State the level of confidence to be used
3. **Solve**
    a. Check that the conditions are satisfied (see guidelines below).
    b. Calculate the level *C* confidence interval for μ

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

where t^* is the upper (1-*C*)/2 critical value for the t distribution on $n-1$ degrees of freedom (using the *t*-applet).

4. **Conclude** in the context of the **stated** problem.

Note: This test can be carried out using the one-sample or paired-samples *t*-statistics procedure in SPSS.

Guidelines for use of the *t* procedures

- SRS is the key assumption; more important than the Normality assumption except in small samples.
- Rough sample size guidelines:
    - $n < 15$: *t* is ok if the data are roughly Normal; if data are skewed or outliers are present, do not use *t*.
    - $15 \le n < 40$: *t* is ok except in the presence of outliers or strong skewness.
    - $n > 40$: *t* is ok even for very skewed distributions (but outliers can always cause trouble.)
- The *t* procedures can be used with data from matched pairs studies by first taking differences.
    - A test of "no change" uses $H_0 : \mu_D = 0$ where μ_D represents the average change.