Chapter 3: The Normal Distributions
Key Ideas

Density curve: a curve that describes the overall pattern of a distribution. It has the following characteristics:

- It is always on or above the horizontal axis;
- It has area exactly 1 underneath it.
- The area under the curve and above any range of values is the proportion of the distribution that falls in that range.

Normal curves are an important class of density curves that are symmetric, single-peaked, and bell-shaped.

The 68-95-99.7 rule for Normal distributions: For the Normal distribution with mean μ and standard deviation σ:

- 68% of the observations fall within σ of μ.
- 95% of the observations fall within 2σ of μ.
- 99.7% of the observations fall within 3σ of μ.

Working with the standard Normal distribution

- **Standardizing and *z*-Scores**

  If x is an observation from a distribution that has mean μ and standard deviation σ, the *standardized value* of x is

$$z = \frac{x - \mu}{\sigma}.$$

  A standardized value is often called a *z*-score.

- **The Standard Normal Distribution**

  The *standard normal distribution* is the normal distribution $N(0,1)$ with mean 0 and standard deviation 1.

  If a variable x has any normal distribution $N(\mu, \sigma)$ with mean μ and standard deviation σ, then the standardized variable

$$z = \frac{x - \mu}{\sigma}.$$

  has the standard normal distribution.

- **Finding Normal Proportions**

  1. State the problem in terms of the observed variable x. Draw a picture to show the area of interest under the normal curve.

  2. Standardize x to restate the problem in terms of a standard normal variable z. Draw another picture to show the area under the *standard* normal curve.

  3. Find the required area under the standard normal curve using technology (e.g., the Rossman/Chance applet at http://www.rossmanchance.com/applets/NormalCalcs/NormalCalculations.html).

  Using technology to compute probabilities, step 2 may not be necessary but it is helpful for getting an intuitive understanding of the proportions.

- **"Backward" Normal Calculations**

  1. State the problem in terms of the observed variable. Draw a picture to show how the desired number x is related to the area of interest under the normal curve.

  2. Draw another picture to show the corresponding area under the *standard* normal curve.

  3. Use technology (e.g., the Rossman/Chance applet) to find the z-value that corresponds to the number x of interest.

  4. Unstandardize z to transform the solution from the z back to the original x scale using

$$x = \mu + z\sigma$$