

Practice Exam II  
(75 points total)

1) (30 points) The random variable  $X$  denotes the time in seconds taken for a file to be downloaded from the web. It is known that  $X$  has a Normal distribution with mean  $\mu = 65$  seconds and standard deviation  $\sigma = 12$  seconds.

a) Write the event "the file takes less than 1 minute to download" in terms of  $X$ .

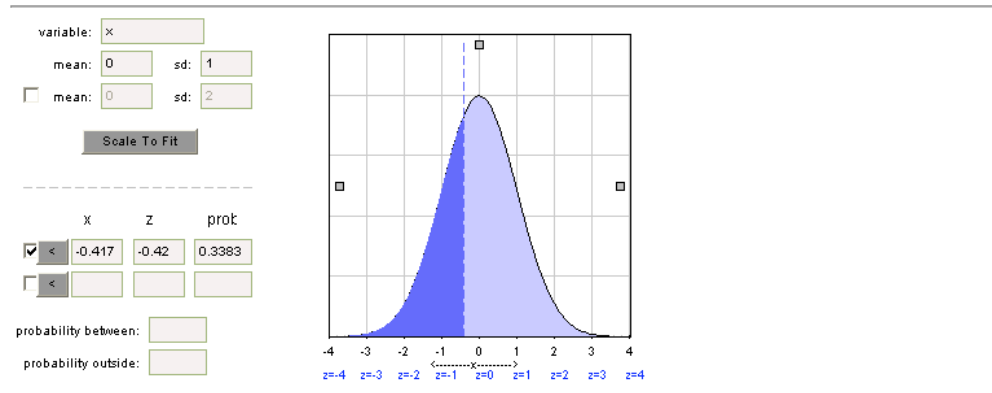
Ans:  $X \leq 60$ .

b) Find the probability of this event. Show your computations.

Ans:  $P(X \leq 60) = P\left(Z \leq \frac{60 - 65}{12}\right) = P(Z \leq -.417) = \boxed{.3383}$

Rossman/Chance Applet Collection

Normal Probability Calculator



c) In a single day, the file is downloaded 50 times. Assuming the download times to be a simple random sample of all download times, what is the sampling distribution of the mean of the 50 times?

Ans: The sampling distribution of  $\bar{X}$  has mean  $= \mu = 65$ , standard deviation =

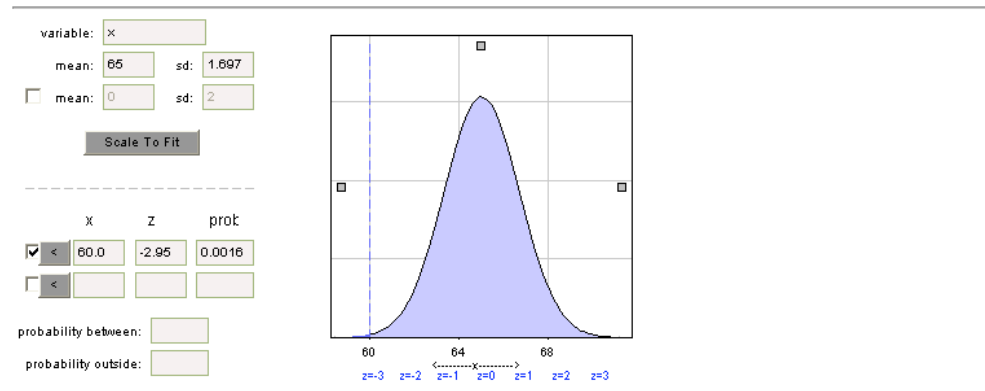
$\frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{50}} = \boxed{1.697}$ , and normal shape by the central limit theorem.

- d) What is the probability that the average of the 50 download times is less than one minute? Show your work.

*Ans: Using the Rossman-Chance applet with mean=65 and sd=1.697, we find the probability to be .0016.*

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### Normal Probability Calculator

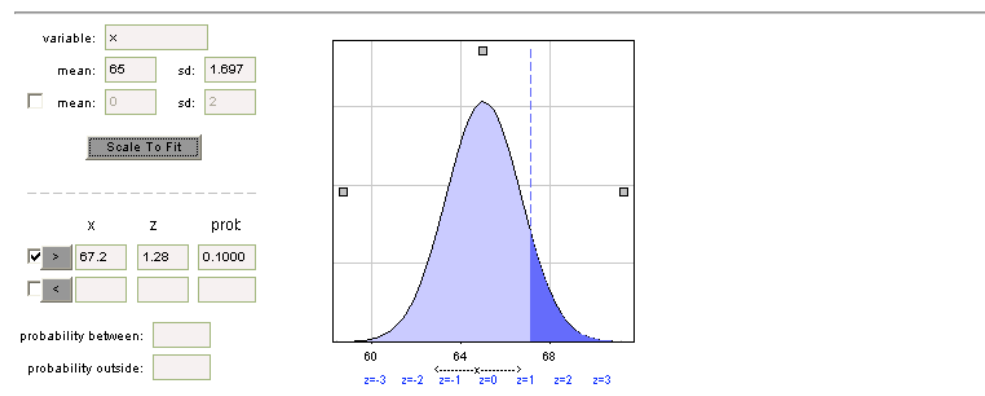


- e) Find the level  $L$  such that there is probability only 0.10 that the mean  $\bar{x}$  of the 50 download times falls above  $L$ .

*Ans: To have the property that there is probability only 0.10 above it, the mean would have to be  $z^* = 1.28$  (use Rossman-Chance Applet) standard deviations above its mean. In this case, we have already seen that the standard deviation of  $\bar{X}$  is 1.697 so the answer is  $L = 65 + 1.28(1.697) = 67.17$ . The Rossman-Chance applet may also be used directly as depicted below:*

## Rossman/Chance Applet Collection

### Normal Probability Calculator



f) Suppose that download times were, in fact, not Normal but were skewed right. Would the probability requested in d) be affected by the skewness? Briefly explain.

*Ans: No the probability would not be materially affected since the Central Limit Theorem would ensure that the sampling distribution of  $\bar{X}$  would still be Normal since the sample size is large ( $n=50$ ).*

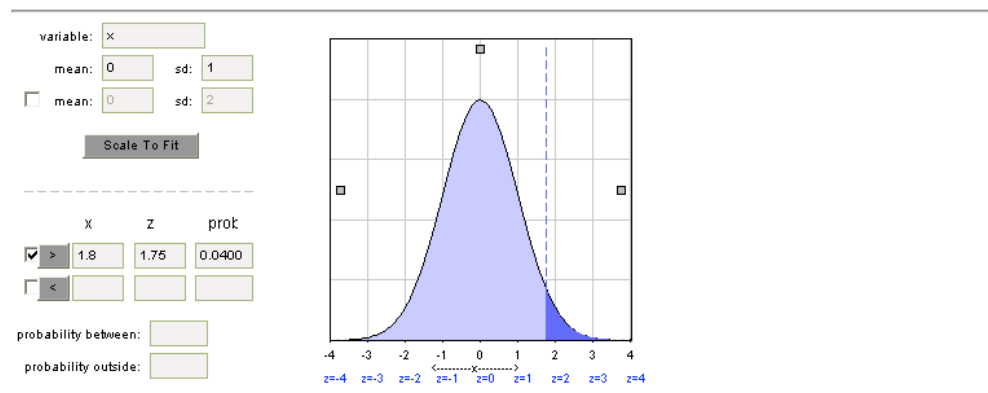
2) (20 points) The distribution of blood cholesterol level in the population of young men aged 20 to 34 years is close to Normal with standard deviation  $\sigma = 41$  milligrams per deciliter (mg/dl).

a) You measure the blood cholesterol of 14 male cross-country runners. The mean level is  $\bar{x} = 164$  mg/dl. Assuming that  $\sigma$  for male cross-country runners is the same as in the general population of men aged 20 to 34 and that the cholesterol levels have a Normal distribution, compute a 92% confidence interval for the mean level  $\mu$  among cross-country runners. Show computations including a picture like those we've used in class to show how you found  $z^*$ .

*Ans: Below is the required picture with area .04 in the upper tail so  $z^* = 1.75$ . Hence, the required 92% confidence interval is  $\bar{X} \pm z^* \frac{\sigma}{\sqrt{n}}$  or  $164 \pm 1.75 \frac{41}{\sqrt{14}}$  which gives (144.82, 183.18).*

## Rossman/Chance Applet Collection

### Normal Probability Calculator



b) What condition not mentioned in a) is required for your computed interval to be valid?

*Ans: We are not told whether the 14 runners are a simple random sample from the population of interest.*

c) A student interprets 92% confidence as follows below. Explain briefly but clearly in what way this explanation is flawed.

"If cholesterol levels were measured in 100 random samples of cross country runners, the intervals obtained from 92 of the samples would contain the mean of our sample."

*Ans: The problem is in the last phrase, "would contain the mean of our sample." Instead, this phrase should be "would contain the mean of the population."*

**3) (25 points)** Continue to consider inference about cholesterol levels introduced in the previous problem. The mean blood cholesterol level for all men aged 20 to 34 years is  $\mu = 188$  mg/dl.

a) Use the results of problem 2a) to test whether the mean for cross-country runners is lower than for the population of men aged 20 to 34 years. State hypotheses, carry out an appropriate test of significance, give the  $P$ -value and state your conclusion. Be sure you give a careful one-sentence interpretation of what the  $p$ -value means in this context.

*Ans: Let  $\mu$  represent the mean cholesterol level of male cross-country runners. Then, our hypotheses are:*

$$H_0 : \mu = 188$$

$$H_a : \mu < 188$$

*That is, the null hypothesis states that cross-country runners have a mean cholesterol level that is the same as that of men aged 20 to 34 years versus the alternative the cross-country runners have a lower mean cholesterol level than men 20 to 34. We compute the test statistic and find the  $p$ -value as follows:*

$$Z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{164 - 188}{41/\sqrt{14}} = \boxed{-2.19}$$

$$p\text{-value} = P(Z < -2.19) = \boxed{.0143}$$

*We have moderate to strong evidence against the null hypothesis in favor of the alternative that male cross-country runners have lower average cholesterol levels than the general population of men aged 20 to 34.*

b) Briefly explain why the observed sample mean of  $\bar{x} = 164$  is stronger evidence against the null in favor of the alternative hypothesis than a sample mean of  $\bar{x} = 174$  would have been. (Note: You should not make any computations to answer this question.)

*Ans: If the null hypothesis were really true and cross country runners actually had average cholesterol levels of 188 mg/dl, a sample mean of 164 would be less likely to occur just due to random sampling variability than would 174. Since it would be less likely if the claim were true, it is stronger evidence that the claim is not true.*