

Relationships of Changes in Price, Price Elasticity and Total Revenue

1. By definition, total revenue (TR) is obtained by multiplying quantity demanded of a product (Q_x) by price (P_x), that is,

$$TR = Q_x \cdot P_x. \quad (1)$$

In class, by taking the derivative of the above total revenue equation with respect to price (dTR/dP_x), we obtain the following general functional relation:

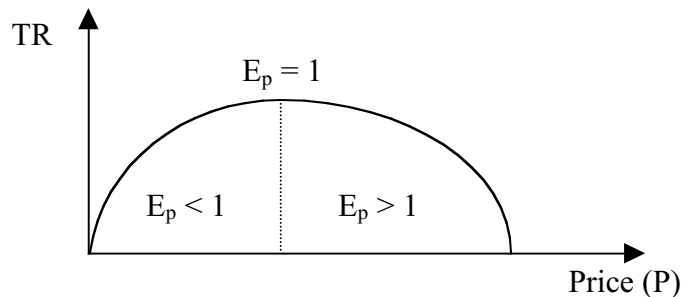
$$dTR/dP_x = Q_x (1 + E_p) \quad (2).$$

In Equation (2), E_p represents the price elasticity of demand. Since E_p is always negative, Equation (2) can be modified as follows:

$$dTR/dP_x = Q_x (1 - |E_p|) \quad (3).$$

Based on Equation (3) we can deduce the following general relationships between a change in total revenue and price, and price elasticity of demand.

- a) If $|E_p| > 1$, then $dTR/dP_x < 0$. In plain English, this says that when demand is price elastic, the relationship between price and total revenue is negative. That is, an increase in price will decrease total revenue and a decrease in price will have the opposite effect on total revenue.
- b) If $|E_p| < 1$, then $dTR/dP_x > 0$. Again, in plain English, this says that when demand is price inelastic, the relationship between price and total revenue is positive. That is, an increase in price will have the effect of increasing total revenue and a decrease in price will cause a decline in revenue.
- c) If $|E_p| = 1.0$, then $dTR/dP_x = 0$. Thus, a change in price will have no effect on total revenue.



2. Let us illustrate the above relationship using the following explicit demand equation:

$$Q_x = 100 - 5P_x, \quad (4a)$$

or,

$$P_x = 20 - 1/5 Q_x \quad (4b)$$

Thus, using Equation (1) and (4b) together, the equation for total revenue can be derived as,

$$TR = Q_x \cdot P_x = Q_x (20 - 1/5 Q_x) = 20Q_x - 1/5 (Q_x)^2$$

Table 1: Relationships of Changes in Price, Total Revenue and Price Elasticity.

Price (P_x)	Demand or $Q_x = 100 - 5P_x$	Total Revenue or $20Q_x - 1/5 (Q_x)^2$	Elasticity or $E_p = (dQ_x/dP_x) \cdot P_x/Q_x$	Elasticity
2	90	180	-0.11	inelastic
4	80	320	-0.25	inelastic
6	70	420	-0.43	inelastic
8	60	480	-0.67	inelastic
10	50	500	-1.00	unitary elastic
12	40	480	-1.50	elastic
14	30	420	-2.33	elastic
16	20	320	-4.00	elastic
18	10	180	-9.00	elastic
19	5	95	-19.00	elastic

The following three observations can be made from the above table:

- In the region of the demand schedule where the absolute value of price elasticity is less one (inelastic), an increase in price is associated with an increase in total revenue and vice versa.
- In the region of the demand schedule where the absolute value of price elasticity is unitary, total revenue is at its maximum.
- In the region of the demand schedule where the absolute value of price elasticity is greater than 1, an increase in price would lead to a decrease in revenue and a decrease in price will have the opposite effect.